

# Tax Morale and Tax Evasion: Social Preferences and Bounded Rationality

August 2, 2011

Zsombor Z. Méder<sup>1</sup>, András Simonovits<sup>2</sup>, and János Vincze<sup>3</sup>

<sup>1</sup> Maastricht University, Department of Economics  
P.O. Box 610, 6200 Maastricht, the Netherlands  
mederzsombor@gmail.com

<sup>2</sup> Institute of Economics, Hungarian Academy of Sciences  
Budaörsi út 45, 1112 Budapest, Hungary  
simonov@econ.core.hu

<sup>3</sup> Corvinus University of Budapest  
Fővám tér 6-8, 1093 Budapest, Hungary  
janos.vincze@uni-corvinus.hu

**Abstract.** We study a family of models of tax evasion, where a flat-rate tax finances only the provision of public goods. Workers have equal income, and they decide how much to report. Individual utility is the sum of utility from private consumption and of moral utility. The latter is a function of three factors: exogenous tax morale, average reported income observed in the neighborhood and private reports. It can be shown that the stability of the equilibrium of reports and the asymptotic convergence rate are independent of the size of the neighborhood. We also create an agent-based model of tax evasion, where we obtain and monotonic Laffer-curves.

## 1 Introduction

Models of tax compliance are closely related to public good provision problems. Inherent in these models is an incipient contradiction between individual rationality and social optimum. Individual rationality calls for tax evasion (free-riding), if it is costless, since only a small fraction of every euro one pays in taxes is returned to him by way of public services. Therefore tax evasion has been regarded as an incentive problem. If monitoring is costly, then it is a nontrivial issue to determine optimal audit probabilities and penalty schemes. The first mathematical analyses, Allingham and Sandmo (1972) and Yitzhaki (1974), modeled tax evasion as a gamble: for given audit probability and a penalty proportional to the evaded tax, what share of their income do risk averse individuals report?

Subsequent studies have discovered that the actual probability of audits and the penalty rate are insufficient to explain why citizens of some societies pay income taxes to such a high extent they do. A parallel development was the repeated finding of non-Nash behavior in experimental public good provision

and related games. In experimental situations the usual finding is substantial positive contribution. This is a robust outcome in experimental ultimatum, dictatorship or trust games selfish individual rationality is consistently refuted, and some sort of “social behavior” prevails. Though in iterated experiments of Public Good games the level of apparent “benevolence” is diminishing, it is not obvious whether this is due to learning or dissatisfaction with others’ “asocial” behavior, see Fehr and Camerer (2004). Apparently people have other motives than maximizing their own incomes, since these experiments have such a simple structure that it is very unlikely that cognitive limitations could explain the altruism exhibited by participants. If we abstract away from audits and penalties, tax compliance is like a public-good provision game, including income redistribution. Public Good provision games belong, in their turn, to the family of Prisoner’s Dilemma games, with many players. It is not surprising that the first explanation that occurred to researchers was to extend the concept of utility, and to invoke some sort of social preferences. In the tax evasion literature this amounted to the introduction of *tax morale*, as in Frey and Weck-Hannemann (1984) and as in the latest verification Lago-Penas and Lago-Penas (2010). The experimental findings corroborate the important role of tax morale, including the impact of social relationships, as moral behavior may not be independent of the observation of our neighbors’ acts, see Heinrich et al. (2004a), Janssen and Ahn (2003), Cason, Saijo and Yamato (2002).

To study the role of social preferences in isolation Simonovits (2010) took tax morale as given, assumed a utility function which took into account the utility derived from reporting income along the utility derived from his own consumption. Specifically, utility increases with tax morale. To avoid perverse cases, Simonovits assumed such a parametric utility function, which excludes overreporting of income. In his model, taxes finance income redistribution as well as provision of public goods which also generates utility. Obviously, with sufficiently large population, optimizing individuals will not take the latter term into consideration. The optimal report achieves a balance between the larger consumption due to lower report and the moral utility arising from higher report. The major result, so far substantiated only by numerical examples, is that higher tax morale induces greater income redistribution and net tax, financing more public expenditures.

Traxler (2010) used a more refined model, making the moral utility dependent on the share of norm-followers: the less people cheat, the higher is the moral utility of reporting income. He also incorporated the auditing and penalizing block from Allingham and Sandmo (1972). Now the existence of an equilibrium is quite involved and even the existence of multiple equilibria cannot be ruled out.

Another branch of the literature has used the agent-based approach to taxation problems, see Bloomquist (2006) or Szabó, Gulyás and Tóth (2009) for a different flavor. The agent-based approach is justified by the complexity of phenomena that resist a purely analytical approach. Another basic insight of the literature is that tax behavior is heterogeneous and cannot be described by

simple “selfish” utility maximization. Several papers are characterized by multi-type taxpayers. Hokamp and Pickhardt (2010) is a good example, where there are four types: a rational (traditional selfish utility maximizer), another is a moralist (who pays all his dues on principle), an erratic type (representing, for example, those who overreport their income) and an emulator (who imitates the behavior of others within a social network). Similar models are offered by Antunes, Urbano, Moniz and Roseta-Palma (2006), Frey and Torgler (2007), Prinz (2010). These models do not assume social preferences as such. Agents are equipped with simple strategies (behavioral rules), and sometimes change their behavior by adopting better performing strategies. “Preferences” are selfish, i.e. agents do not care directly for others’ happiness, however, due to bounded rationality, they may behave as if they did. These models usually focus on the problem of punishing tax evaders.

In this paper we follow both branches of the literature, and try to make qualitative comparisons. To sharpen our focus we neglect audits and fines, like Simonovits (2010). Thus we can reformulate our main question as follows: what are the consequences of different views on human behavior (utility maximizers with social preferences or boundedly rational non-optimizing agents) in the Utopian world where nobody is forced to contribute to social funds?

For the sake of simplicity, we neglect pre-tax wage differences as well as income redistribution. Our tax system works with a flat rate without cash-back, therefore we shall drop the adjective marginal from now on. In models A and B, agents maximize utility functions with social preferences. In model A, every worker knows the average tax report of the population. In model B every worker has his own time-invariant neighborhood and he is able to observe his neighbors’ tax reports. In both cases we can investigate the existence of an equilibrium or multiple equilibria, and their stability.

Analyzing models A and B, we speak of a symmetric case if the exogenous tax morales are identical. Then, there exists a unique symmetric equilibrium, which is stable in the case of full population observation, and remains stable for partial observation as well. Garay, Simonovits and Tóth (2010) gives an analytical proof in a more general framework. Given the individual utility functions, the government can maximize its social welfare function by choosing the tax rate. The Laffer-curve generated by the model has an inverse U-shape.

In our agent-based model (C) agents make cue-induced decisions and either report their full income, or evade taxes completely. There are two heuristics: one based on material satisfaction, the other on the behavior of neighbors. Similarly to the first models, there is no external punishment. We focus on the effects of different parameters – tax rate, efficiency of public good provision and social cohesion – on the final levels of tax evasion. We find that, counter-intuitively, social cohesion decreases tax compliance, but is only relevant for low values of efficiency.

The structure of the paper is as follows: Section 2 outlines the models, Section 3 presents the results of the simulation. Section 4 concludes.

## 2 The Models

We shall apply three approaches. a) The traditional neoclassical one, where individuals maximize their morale-dependent utility functions, decide on their optimal reports; but the utility functions also depend on the average reports (proportional to the public expenditures) of the previous period. b) The bounded rationality assumption, where the individuals only observe their narrow neighborhoods, but still optimize their utility functions. c) Not only the neighborhood is narrow but the decision is also boundedly rational. We shall present the three approaches one after another.

### 2.1 Model A: Full observation, utility maximization

Let the number of workers be a positive integer  $I$  and index the workers as  $i = 1, 2, \dots, I$ . Let the index of periods (say years) be  $t = 0, 1, 2, \dots$ ,

We assume that every worker has the same earnings, for simplicity, unity:  $w = 1$ . Then there is no reason for income redistribution, the tax only finances the provision of public goods.

Let  $m_i$  be worker  $i$ 's *exogenous tax morale*, let  $v_{i,t}$  be his income report in period  $t$ ,  $v_m \leq v_i \leq 1$ , where  $v_m$  is the minimal report,  $0 \leq v_m < 1$ . (By the logic of the theory, the government also knows that everybody earns a unity, nevertheless, it tolerates underreporting, it only insists on a minimal report.)

In period  $t$  the average reported wage is equal to

$$\bar{v}_t = \frac{\sum_{i=1}^I v_{i,t}}{I}.$$

Let  $\theta$  be a real number between 0 and 1, the tax rate. Let  $c_{i,t}$  denote the  $i$ 's worker consumption:  $c_{i,t} = 1 - \theta v_{i,t}$ , its traditional utility function is then  $u(c_{i,t})$ . Let the moral utility function be  $z(v_{i,t}, m_i, \bar{v}_{t-1})$ , which has three components: utility derived from his own report  $v_{i,t}$ , his exogenous morale  $m_i$  and  $\bar{v}_{t-1}$ . (The exogenous tax morality has no direct economic meaning, it can only be inferred from indirect observations.) Finally let the per capita public expenditure be  $X_t$ , i.e.  $X_t = \theta \bar{v}_t$ , whose individual utility is  $q(X_t)$ .

Let us assume that  $u(\cdot)$ ,  $z(\cdot, \cdot, \cdot)$  and  $q(\cdot)$  are strictly increasing and strictly concave. The worker  $i$ 's utility is the sum of three terms:

$$U_{i,t}^* = u(c_{i,t}) + z(v_{i,t}, m_i, \bar{v}_{t-1}) + q(X_t).$$

Of course, maximizing  $U_i$ , the worker neglects the third term, because this depends on the simultaneous decisions of many other workers. In other words, in period  $t$  worker  $i$  reports such an income which maximizes his narrow utility:

$$U_{i,t}(v_{i,t}) = u(1 - \theta v_{i,t}) + z(v_{i,t}, m_i, \bar{v}_{t-1}) \rightarrow \max.$$

In case of interior optimum,

$$U'_{i,t}(v_{i,t}) = -\theta u'(1 - \theta v_{i,t}) + z'_1(v_{i,t}, m_i, \bar{v}_{t-1}) = 0,$$

where  $z'_1$  is the partial derivative of function  $z$  with respect to the first variable.

In case of a corner solution,  
either

$$v_{i,t} = v_m \quad \text{if} \quad U'_{i,t}(v_m) = -\theta u'(1 - \theta v_m) + z'_1(v_m, m_i, \bar{v}_{t-1}) < 0,$$

or

$$v_{i,t} = 1 \quad \text{if} \quad U'_{i,t}(1) = -\theta u'(1 - \theta) + z'_1(1, m_i, \bar{v}_{t-1}) > 0.$$

The system's initial state is  $\mathbf{v}_0 = (v_{1,0}, \dots, v_{I,0})$ . The system is in an equilibrium, if starting the system from it, the system remains there. Notation:  $\mathbf{v}^o = (v_1^o, \dots, v_I^o)$ .

The system is called asymptotically stable if (i) for any real  $\varepsilon > 0$  there exists another real  $\delta > 0$ , such that if the initial state is in the  $\delta$ -neighborhood of the equilibrium, it stays in the  $\varepsilon$ -neighborhood of the equilibrium and (ii)  $v_{i,t}$  asymptotically converges to  $v_i^o$ .

The social welfare function is given by

$$V = \sum_{t=0}^{\infty} \beta^t \sum_{i=1}^I U_{i,t},$$

where real  $\beta$  is between 0 and 1, the discount factor.

We need apply parameterized rather than general utility functions, c.f. Simonovits (2010).

$$u_i(c_{i,t}) = \log c_{i,t}, \quad z(v_{i,t}, m_i, \bar{v}_{t-1}) = \bar{v}_{t-1} m_i (\log v_{i,t} - v_{i,t}), \quad q(X_t) = \omega \log X_t,$$

where real  $\omega > 0$  is the efficiency parameter of the public expenditures. Note that the first and second factors of  $z$  are endogenous and exogenous tax morales, respectively. The third factor, in brackets, increases with reported income  $v_{i,t}$  for  $v_{i,t} < 1$  and decreases otherwise: there is no moral urge to overreport income.

Then the implicit difference equation of the optimal report is

$$-\frac{\theta}{1 - \theta v_{i,t}} + \bar{v}_{t-1} m_i \left( \frac{1}{v_{i,t}} - 1 \right) = 0, \quad i = 1, 2, \dots, I.$$

Making it explicit, and solved,

$$v_t = \frac{\theta + m_i(\theta + 1)v_{t-1} - \sqrt{[\theta + m_i(\theta + 1)v_{t-1}]^2 - 4m_i^2\theta v_{t-1}^2}}{2m_i\theta v_{t-1}}.$$

Finally, it is worth displaying the equation of the equilibrium, at least for the symmetric case  $m_i = m$ :

$$\frac{-\theta}{1 - \theta v^o} + v^o m \left( \frac{1}{v^o} - 1 \right) = 0.$$

Then  $c^\circ = 1 - \theta v^\circ$ ,  $z^\circ = v^\circ m \log v^\circ$  and  $q(X^\circ) = \omega \log(\theta v^\circ)$ .

It can be shown that the symmetric equilibrium of reports is the lower root of a quadratic equation:

$$v^\circ = \frac{1 + \theta - \sqrt{(1 - \theta)^2 + 4\theta^2/m}}{2\theta}.$$

It is easy to see that the equilibrium report is an increasing function of the exogenous morale. It is more difficult to prove that the equilibrium report is a decreasing function of the tax rate.

When does an equilibrium exist? If  $v_m \leq v^\circ \leq 1$ , i.e.

$$-\frac{\theta}{1 - \theta v_m m} + v_m m \left( \frac{1}{v_m} - 1 \right) > 0, \quad \text{i.e.} \quad m > \frac{\theta}{(1 - \theta v_m)(1 - v_m)}$$

In principle, we can determine the socially optimal tax rate which maximizes  $V$ , or  $U_i$ :

$$U_i(\theta) = \log(1 - \theta v^\circ(\theta)) + v^\circ(\theta) m (\log v^\circ(\theta) - v^\circ(\theta)) + \omega \log(\theta v^\circ(\theta)).$$

At this point we introduce the famous *Laffer-curve*, which depicts the tax revenue as a function of the tax rate:

$$X(\theta) = \theta v^\circ(\theta).$$

It can be shown that the Laffer-curve is increasing for low tax rates and decreases for high rates assuming  $m < 6 + 2\sqrt{5} \approx 10.5$ .

Returning to the dynamics, note that after one step, the non-equilibrium path also becomes symmetric:  $v_{i,t} = v_t$ , because everybody will have the same optimization problem. Dropping index  $i$ ,

$$F(v_{t-1}, v_t) = -\frac{\theta}{1 - \theta v_t} + v_{t-1} m \left( \frac{1}{v_t} - 1 \right) = 0.$$

The sufficient condition of local asymptotic stability is  $|F'(v^\circ)| < 1$ , where by the theorem on implicit function,

$$\lambda = \frac{dv_t}{dv_{t-1}} = -\frac{F'_1}{F'_2}.$$

Here  $F'_1$  and  $F'_2$  are the partial derivatives with respect to variable 1 and 2, respectively.

$$F'_1 = m \left( \frac{1}{v^\circ} - 1 \right) \quad \text{and} \quad F'_2 = \frac{-\theta}{(1 - \theta v^\circ)^2} + m \frac{1}{v^\circ}.$$

With arrangement, the local stability condition is

$$0 < \theta < m(1 - \theta v^\circ)^2$$

The issue of global stability requires further analysis.

## 2.2 Model B: Limited Observation, Utility Maximization

We replace now the assumption of model A about the global observability by the local observability. Let  $N_i$  be worker  $i$ 's neighborhood (the index set of its neighbors), which is a small subset of the total index set  $N = \{1, 2, \dots, I\}$ . Let  $\bar{v}_{i,t}$  be the average report of  $i$ 's neighborhood in time  $t$ :

$$\bar{v}_{i,t} = \frac{\sum_{j \in N_i} v_{j,t}}{|N_i|},$$

where  $n_i = |N_i|$  is the number of neighbors, a small number relative to  $I$ , and in the symmetric case independent of  $i$  and  $n_i \ll I$ . We assume that the system of neighborhoods is connected, i.e. any pair of individuals are connected by a chain of neighbors. Moreover, there is an integer  $T$ , ( $T \leq I$ ) that any pair is connected by a chain of length  $T$ .

Because of this generalization, we display several formulas again, in the general frame:

$$U'_{i,t}(v_{i,t}) = -\theta u'(1 - \theta v_{i,t}) + z'_1(v_{i,t}, m_i, \bar{v}_{i,t-1}) = 0,$$

where  $z'_1$  is the partial derivative of  $z$  with respect to the first variable.

In case of corner solution, either

$$v_{i,t} = v_m \text{ if } U'_{i,t}(v_m) = -\theta u'(1 - \theta v_m) + z'_1(v_m, m_i, \bar{v}_{i,t-1}) < 0,$$

or

$$v_{i,t} = 1 \text{ if } U'_{i,t}(1) = -\theta u'(1 - \theta) + z'_1(1, m_i, \bar{v}_{i,t-1}) > 0.$$

Then the equation for the optimal report (for interior optima) is

$$-\frac{\theta}{1 - \theta v_{i,t}} + \bar{v}_{i,t-1} m_i \left( \frac{1}{v_{i,t}} - 1 \right) = 0.$$

We often assume that the model is symmetric, i.e. if arbitrarily permute the individuals' indices, then the neighborhoods are also permuted correspondingly. Specifically, for every  $i$ ,  $|N_i| = n$  and  $m_i = m$ .

For the time being, we consider the simplest case.

**Example 1.** The workers are located on a circle: with convention  $I + 1 = 1$  and  $0 = I$ ,  $N_i = \{i - 1, i + 1\}$ . Then  $\bar{v}_{i,t-1} = (v_{i-1,t-1} + v_{i+1,t-1})/2$ , at an equilibrium  $v_i^o$  as in model A

To study local stability, we shall return to the general case, where the entries of the matrix of the linearized equation  $\mathbf{v}_t = A\mathbf{v}_{t-1}$  are as follows:

$$a_{ij} = \begin{cases} F'(v^o)/n_i, & \text{if } j \in N_i; \\ 0, & \text{otherwise.} \end{cases}$$

**Theorem.** Under the stated assumptions, the symmetric equilibrium is locally asymptotically stable.

*Proof.* Due to symmetry, the row sums of  $A$  are uniformly equal to  $F'(v^o)$ . By the theory of Markov chains, the underlying Markov system is ergodic, its dominant characteristic root is equal to 1. Hence the dominant characteristic root of  $A$  is equal to  $F'(v^o)$ . Q.E.D.

In a separate paper, Garay, Simonovits and Tóth (2010) showed under quite general assumptions that the equilibrium is unique, symmetric and globally stable.

### 2.3 Model C: Limited Observation, Decisions Based on Simple Heuristics

**Overview** Alternatives to economics-style theories of human decision-making stress the role of simple heuristics and the lack of "integrated" preferences. In this framework people have available a set of heuristics (decision rules), a mechanism for selecting a decision rule and, possibly, a meta-rule that updates either the set of heuristics, or the mechanism performing the selection, see Gigerenzer and Selten (2002). In our model of taxation we assume the existence of two heuristics.

1. Pay taxes if you are satisfied with your lot, and do not pay if you are dissatisfied.
2. Pay taxes if most of your neighbors pay, and do not pay if most of them do not pay.

The first heuristic is based on a cue that is close to the concept of utility. Agents consume private goods and enjoy the fruits of public good provision. These two sources of happiness are integrated into a traditional utility function. Agents are heterogeneous, in the sense that the utility threshold above which they feel satisfied is different amongst them. This property of the model has close resemblance to Simon's concept of satisficing, see Simon (1956). The second heuristic is the well-established imitation heuristic adapted to our problem, having a statistical edge.

We believe that both heuristics are sensible. Paying taxes without the danger of retribution is an act that immediately reduces personal consumption, without increasing perceptibly the enjoyment of public-good services. If we consider tax-paying as a social norm we can assume that people are more likely to follow this norm if they can afford it. It is easy to see that fully unselfish people would be exploited mercilessly, and probably driven out of existence. On the other hand, imitation, even thoughtless imitation, seems to be a general purpose heuristic, or a basic constituent of the human mind (see Meltzoff (1988) who proposed the expression "homo imitans" for our race).

However, these two decision rules can conflict. In the traditional preference-based approach such a problem cannot arise because preferences are integrated, according to the parlance of the psychological literature. In our framework we must postulate some conflict-resolving mechanism, or a meta-heuristic. We propose two meta-rules:

1. Pay taxes if any of the two (first-level) heuristics suggests tax-paying.
2. Pay taxes only if both of the (first-level) heuristics suggest tax-paying.

One cannot say that any of the two meta-rules is more plausible than the other, therefore we wish to have a mechanism that helps agents in adopting one or the other. Our adoption hypothesis is vaguely based on the sociological concept of "cognitive dissonance", see Festinger (1957). Specifically, we consider the first meta-heuristic as tax-paying being default decision, while the second meta-rule takes cheating as the default. We hypothesize that people may change their defaults if they apply the non-default decision for too long. If I consider myself as someone who is a tax-payer in general, but in the last years I did not happen to pay any taxes – because I felt dissatisfied all the time and saw most of my acquaintances behaved in a like manner – then I will probably change my default rather than feel ridiculous.

**Specification** We construct a simple agent-based model to analyze the way local interactions foster or hinder the spreading of tax evasion. There are  $I$  agents, having uniform real income (measured in a "private" good) in each period  $t$ , which is normalized to 1. The individual budget constraints are:

$$c_{it} + \tau_{it} = 1,$$

where  $c_{it}$  denotes private consumption, and  $\tau_{it}$  represents taxes paid. Agents have identical, linear utility functions. We chose these for the sake of simplicity, since qualitative results from runs with logarithmic utility functions were similar. Thus, we kept the simpler form of utility function:

$$u_{it} = c_{it} + \chi X_t,$$

where

$$X_t = \frac{\sum_{i=1}^I \tau_{it}}{I}$$

represents the service of the public good enjoyed by an agent. We chose linear utility for simplicity, but qualitative results for logarithmic utility were similar. The numerator is the amount of the public good financed by tax. Parameter  $\chi$  is introduced to capture the effectiveness of the public sector. Thus if  $\chi > 1$ , then public spending is indeed efficient and the socially optimal outcome would involve a maximal tax rate with full adherence to the taxpaying norm. On the other hand if  $\chi < 1$ , then public waste outweighs the efficiency of producing the public good and in optimum nobody pays taxes, or the tax rate is nil.

Let  $0 < \theta < 1$  be the tax rate. The tax behavior of each agent is represented by a binary variable:

$$\tau_{it} = \theta,$$

if agent  $i$  does not pay taxes in period  $t$ , and

$$\tau_{it} = \theta,$$

if he does.

Each agent  $i$  is characterized by a real number  $\underline{u}_i$  giving his frustration threshold in terms of utility. We say that an agent is "internally" frustrated in  $t$  if  $u_{i,t-1} < \underline{u}_i$ . Also, each agent  $i$  has a neighborhood  $N_i \subset N$ , interpreted as the set of acquaintances whose behavior is observable by  $i$ . Agents observe their neighbors' decisions. Let  $m_{i,t}$  represent the share of tax evaders among agent  $i$ 's acquaintances. Then, this agent is "externally" frustrated in  $t$  if  $m_{i,t-1} > 0.5$ . These two rules nail down the heuristics mentioned above.

Tax behavior, is, in the end, determined through one of the meta-rules. If agent  $i$  is following the first meta-rule – pay taxes if any of the heuristics suggest so – we will say that his default behavior is to pay taxes.

$$\tau_{it} = \begin{cases} \theta & \text{if } u_{i,t-1} \geq \underline{u}_i \quad \text{or} \quad m_{i,t-1} < 0.5 \\ 0, & \text{otherwise} \end{cases}$$

For agents following the second meta-rule, we have the converse:

$$\tau_{it} = \begin{cases} \theta & \text{if } u_{i,t-1} \geq \underline{u}_i \quad \text{and} \quad m_{i,t-1} < 0.5 \\ 0, & \text{otherwise} \end{cases}$$

Variable  $\tau_{it}$  is also the (partly) public state of agent  $i$  in period  $t$ , inasmuch as those people whose neighborhoods contain  $i$  can observe it. The agent characteristic ( $\underline{u}_i$ ) is time invariant, and non-observable. To diminish the significance of the initial pattern of tax compliance, we also introduced "noise" in the system. That is, each turn agents have a fixed chance of randomly switching from tax evasion to compliance or vice-versa. Moreover, we assume that agents may switch types to avoid cognitive dissonance: agents following the first meta-rule, i.e. those whose default is tax-paying switch to the second meta-rule if they evade taxes for long enough. Vice-versa, default tax evaders can become default tax-payers.

Tax compliance models usually include audits and penalties, as means to compel selfish agents to comply. We are well aware that real societies use punishment as an incentive device. Therefore our model is purely theoretical, it asks a simple question: can the social norm of tax compliance prevail in a society that is characterized by the properties described above? To answer this question we run simulations, where we observe the path of aggregate compliance starting from arbitrary initial levels.

### 3 Simulations

#### 3.1 Model A

We start our investigations by experimenting with the parameter pairs  $(m, \theta)$ : Table 1. First, we get rid of those pairs which generate optimal reports below the

minimum or above 1. For lower tax morale ( $m = 0.5$ ), lower optimal tax rates (about  $\theta = 0, 25$ ) are obtained, for higher tax morales ( $m = 1$ ), higher optimal tax rates (about  $\theta = 0.5$ ) are obtained. It is remarkable that for low as well as high tax morales, the Laffer curve is already decreases around the optimum: for  $m = 0.5$ , raising the tax rate from 0.2 to 0.25, the tax revenue drops from 0.11 to 0.1; while for  $m = 1$ , raising the tax rate from 0.45 to 0.5, the tax revenue drops from 0.198 to 0.191, while the welfare increases. Notwithstanding, it is still a paradox that the socially optimal report is a decreasing function of the exogenous tax morale.

**Table 1.** The impact of morale and tax rate

| Morale<br>$m$ | Tax Rate<br>$\theta$ | Report<br>$v$ | Tax<br>$X$ | Utility<br>$U$ | Expansion<br>$\lambda$ |
|---------------|----------------------|---------------|------------|----------------|------------------------|
| 0.5           | 0.20                 | 0.551         | 0.110      | -2.639         | -0.623                 |
| 0.5           | 0.25                 | 0.438         | 0.110      | -2.604         | -0.776                 |
| 0.5           | 0.30                 | 0.333         | 0.100      | -2.647         | -0.885                 |
| 1.0           | 0.45                 | 0.439         | 0.198      | -2.396         | -0.809                 |
| 1.0           | 0.50                 | 0.382         | 0.191      | -2.381         | -0.873                 |
| 1.0           | 0.55                 | 0.329         | 0.181      | -2.384         | -0.919                 |

### 3.2 Model B

For symmetric initial states the difference between models A and B immediately disappears, but it survives for asymmetric initial states. The stability of model A also survives.

We select the pair  $m = 1$  and  $\theta = 0.5$  and analyze the dynamics of asymmetric initial states for three agents. Let  $x = 1.3$ ,  $v_{1,0} = xv^o$ ,  $v_{2,0} = v^o$  and  $v_{1,0} = v^o/x$ . It is sufficient to display the first six state vectors to see the preservation of stability: Table 2.

**Table 2.** Limited observation also stabilizes

| Period<br>$t$ | report 1<br>$v_1$ | report 2<br>$v_2$ | report 3<br>$v_3$ |
|---------------|-------------------|-------------------|-------------------|
| 0             | 0.497             | 0.382             | 0.294             |
| 1             | 0.357             | 0.389             | 0.411             |
| 2             | 0.392             | 0.383             | 0.377             |
| 3             | 0.381             | 0.383             | 0.385             |
| 4             | 0.383             | 0.382             | 0.382             |
| 5             | 0.382             | 0.382             | 0.382             |

### 3.3 Model C

We first present the skeleton of the algorithm used to run the simulations. The algorithm was implemented in the NetLogo language. Then, we describe the choice of parameters for the runs, and finally, we present our main results.

#### The Algorithm

1. Define parameters  $(I, \omega, \theta)$ .
2. Determine the neighborhoods  $N_i \subset N$  for each  $i$ .
3. Generate random  $\underline{u}_i$  values.
4. Generate random tolerance levels  $m_i$ .
5. Determine initial strategies: let  $\tau_{i0} = \theta$  with probability  $s_0$ .
6. Calculate  $u_{i,t}$ .
7. Determine  $\tau_{i,t+1}$ .
8. Let  $t = t + 1$ .
9. Iterate on 6, 7 and 8 until the system becomes stationary.

#### Parameter setup

- We set the number of agents to be  $I = 1000$ .
- The dissatisfaction threshold  $\underline{u}_i$  has a normal distribution with  $\alpha = 2$ , the minimum value being  $x_m = 1$  for the linear model. In the logarithmic model,  $\underline{u}_i$  follows an *inverse* Pareto distribution, with the same  $\alpha = 2$ , and the maximum value  $x_m = 0$ .
- Tolerance levels are uniformly distributed on  $(0, 1)$ .
- We chose the noise parameter to be 0.01, so that on average 1 percent of the population switches strategies randomly every period.
- Type-switching occurs stochastically, following an exponential distribution with  $\lambda = \frac{1}{10}$ , if agents behave contrary to the defaults of their types for more than the threshold of 10 periods.
- We run the simulations until the share of tax evaders stabilizes. The time frame of  $t = 700$  periods proved to be sufficient for reaching a stationary state for all parameter combinations. Because of the presence of noise, we can not expect more than stochastic stability. However, to eliminate effects of random drift, we take the average from the last 50 periods of the macro-variable of interest, namely, the share of tax evaders in the population.
- To provide more robust results, we took the average of ten simulation runs for all parameter combinations.

We examine agents in a *circular* and a *grid* neighborhood. In the former, agents are connected to the previous and next  $d$  agents, with  $d \in [1, 10]$ . On the latter they are located on a grid whose edges are connected, i.e. a two-dimensional torus, having four or eight neighbors, depending on whether they are connected to agents placed diagonally from them. We first present results for the circular environment, then compare it to the grid.

The free parameters of the model are the initial share of tax evaders  $s_0$ , the efficiency of the public sector  $\chi \in [0, 2]$ , the tax rate  $\theta \in [0, 1]$ , and  $d$ , which we interpret as the social cohesion of the society.

Our first finding is that the initial share of tax evaders has no permanent effect, thus, the structural characteristics of the simulated society are sufficient to determine long-run tax evasion. Therefore, we fix  $s_0 = 0.5$ , and we can exclude  $s_0$  from our analysis, and reduce the dimensionality of the problem by one.

**Partial effects** Fig. 2 provides an unclear picture on the impact of tax rate. First of all, the share of tax evaders is within a narrow range, between approximately 33% and 37%. This means that, on average, the tax rate has a rather minor effect. Furthermore, there seem to be two segments where tax rate and evasion are inversely related, while between them evasion increases monotonously – but not linearly. Without public good provision, the rate of tax evaders would obviously rise monotonously on the whole interval. Public goods thus complicate the picture.

As expected, evasion and efficiency are inversely related, see Fig. 3, showing an inverse S-shaped curve with an inflexion point, at  $\chi \sim 0.75$ , after which returns start diminishing on the longer, convex part of the curve. Overall, efficiency seems to have a much greater impact on tax evasion than the tax rate itself; particularly, increasing efficiency from very low to low or medium can as much as half the numbers of tax evaders.

Higher social cohesion increases tax evasion, but with diminishing returns. With  $d = 1$ , tax evasion is at a low average level of 25%. This might be surprising, but it indicates that reliance on the external frustration cue, combined with majority imitation, can induce or enforce opportunistic behavior. In case of the circular neighborhood, we typically see clusters of tax evaders separating groups of taxpayers. Although noise temporarily changes their behavior, they continue to evade taxes for long time periods. Obviously, after sufficient time, they become default tax evaders. When this happens, it becomes increasingly difficult for tax compliance to invade this population, since the local social norm is evasion. This seems to replicate actual social behavior quite precisely.

**Cross-effects** In order to gain a deeper understanding how the parameters affect the outcomes, we examined cross-effects of the parameters. Fig. 5 shows the effect of tax for different social distances on the relationship between tax and evasion. It seems that for a fixed social distance, this relationship is simpler. E.g. for  $d = 1$ , tax evasion decreases monotonously; the most likely explanation for this is that with just one neighbor on either side, it is much more difficult to form stable clusters of evasion. This figure also encodes the diminishing effect of cohesion described in the last section. The fact that for larger  $d$ s, the tax/evaders curve seemingly become more and more complex, still lacks an explanation.

For the common effects of efficiency and social cohesion, we note that the curves with greater  $d$  dominate the ones with lower ones (Fig. 4. For "negative" social efficiency – i.e.  $\chi < 1$ , the relationship is nonlinear, S-shaped, except for

$d = 1$ . However, for  $\chi > 1$ , the curves collude into a single linear segment. Thus, in an efficient society, social proximity has little to no effect, and cohesion is more important where public funds are spent inefficiently. On the inefficient segment, the inflexion point is also around  $\chi \sim 0.75$ .

Fig. 6 presents a "heat map" of tax evasion for the two crucial parameters, tax rate and efficiency. The main message here is rather clear: with a low tax rate,  $\theta < 0.15$ , efficiency has but a marginal effect. When taxes are sufficiently high, even with efficiency lower than unity,  $\chi < 1$ , the share of tax evaders is kept at a low. The isocurves indicate a quadratic relationship between tax and efficiency, reflecting the properties of the utility function.

**Laffer-curves** The surprisingly low effect of tax rate on evasion obviously imply increasing Laffer-curves, so Fig. 7 might not be so shocking anymore, with linear Laffer-curves for any fixed efficiency. Efficiency simply determines the slope of the curve. Thus, when efficiency is low, raising taxes can have minimal effect on tax income, while increasing efficiency can do the trick. An interesting phenomenon is the existence of an interval where the effect of efficiency increases rapidly for  $\chi \in [0.6, 0.8]$ . Moving from very low to low efficiency can mean a much lower increase in tax revenues than moving up in the medium-low range. Focusing on efficiency can emerge as best practice for those countries that experience corruption that is not too high.

**Circular vs. grid environment** Meaningful comparison between the grid and circular neighborhood structures can be made in the cases with  $d = 2$  and  $d = 4$  for the circular environment, since in these cases the number of neighbors for each agent is the same as in the grid without or with diagonally placed agents being neighbors. There are a number of differences between the two graphs, most notably the maximal and the average distance between two random agents being much greater in the circular environment. Moreover, a grid is much less clustered, the neighbors of an agents' neighbors are typically not connected to that agent. Therefore, it is expected that clusters of tax evaders are less likely to form.

It is easy to find evidence for this phenomenon. Averaging over all variables, the grid environment allows for a lower level of tax evasion (29.1%) than the circular one (34.4%). Fig. 8 provides a heat map on the difference in evasion for fixed tax-efficiency combinations. When the tax rate – and thus tax evasion – is low, the circular environment performs better. But when the tax rate is increased, the advantages of the grid environment become more accentuated, particularly in the critical zone of  $\chi \in [0.5, 0.8]$ . While tax evasion surges in the circular environment in this zone, its spread is hindered by the more vulnerable neighborhood structure of the grid environment.

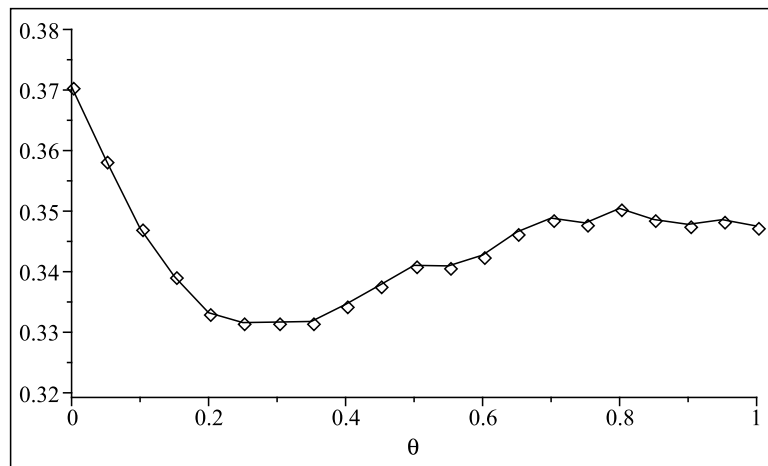
## 4 Conclusions

We have analyzed the three models of reporting income: A) global observation and utility maximization; B) local observation and utility maximization; C) local observation and decisions based on simple heuristics. Models A and B are too similar to each other, and the difference between global and local does not change the qualitative results. The results obtained in the agent-base model complement those derived above: it turns out that the degree of tax evasion is increasing with the size of the neighborhood. The unexpected relationship between the tax rate and tax evasion in this model requires further investigation through agent-based models.

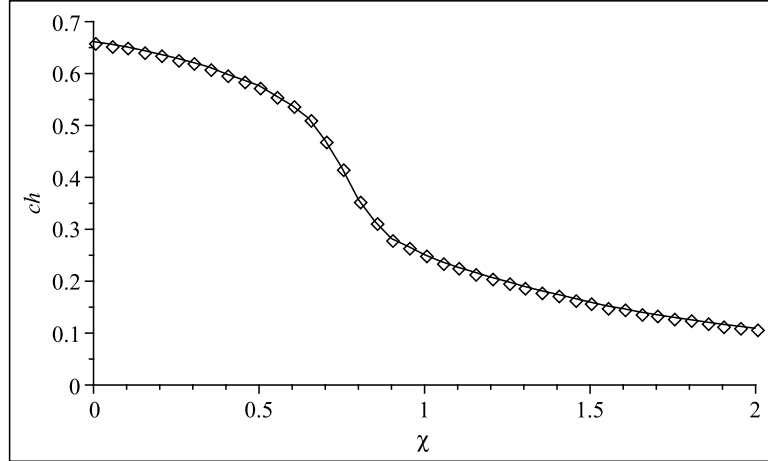
## Figures

**Fig. 1.** Laffer-curves

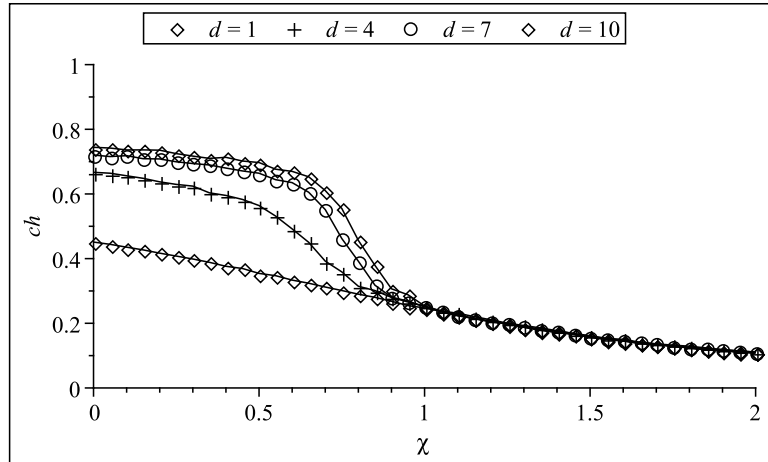
**Fig. 2.** The average impact of tax rate on tax evasion



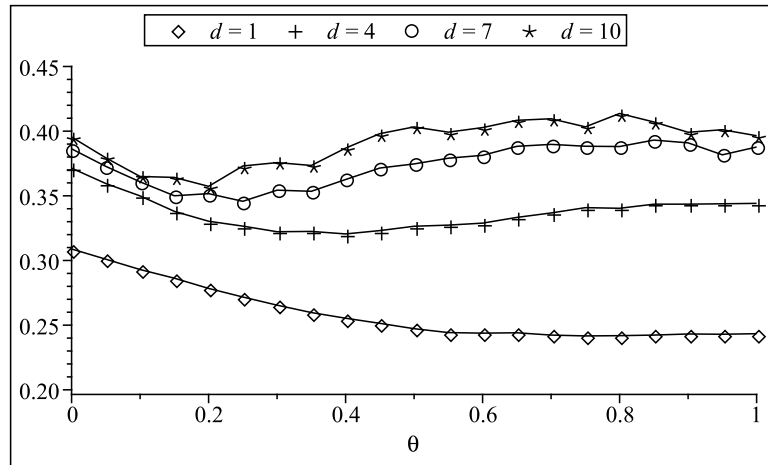
**Fig. 3.** The average impact of public efficiency on tax evasion



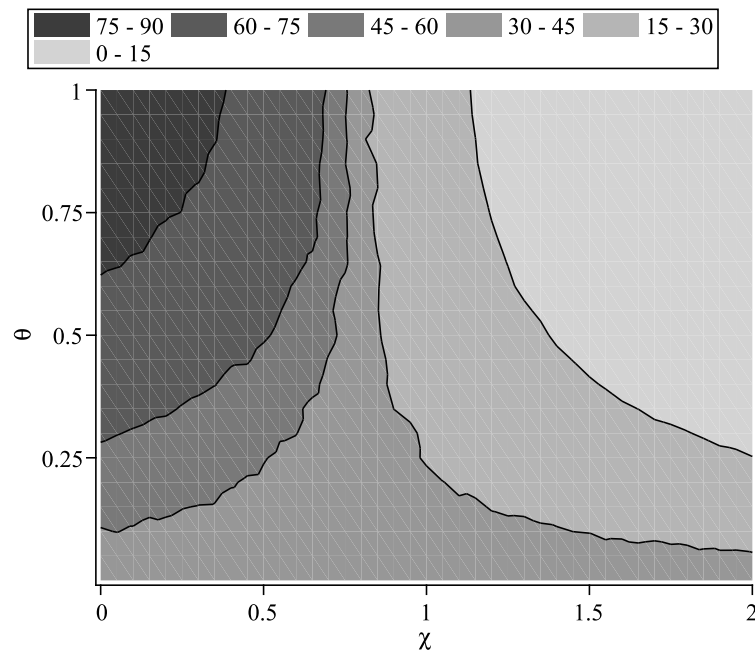
**Fig. 4.** The average impact of public efficiency on tax evasion for different values of social cohesion



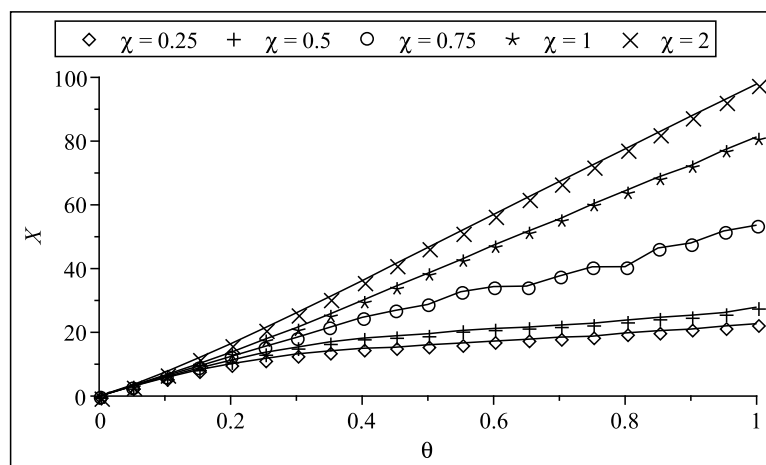
**Fig. 5.** The average impact of the tax rate on tax evasion for different values of social cohesion



**Fig. 6.** The average impact of the public efficiency and the tax rate on tax evasion



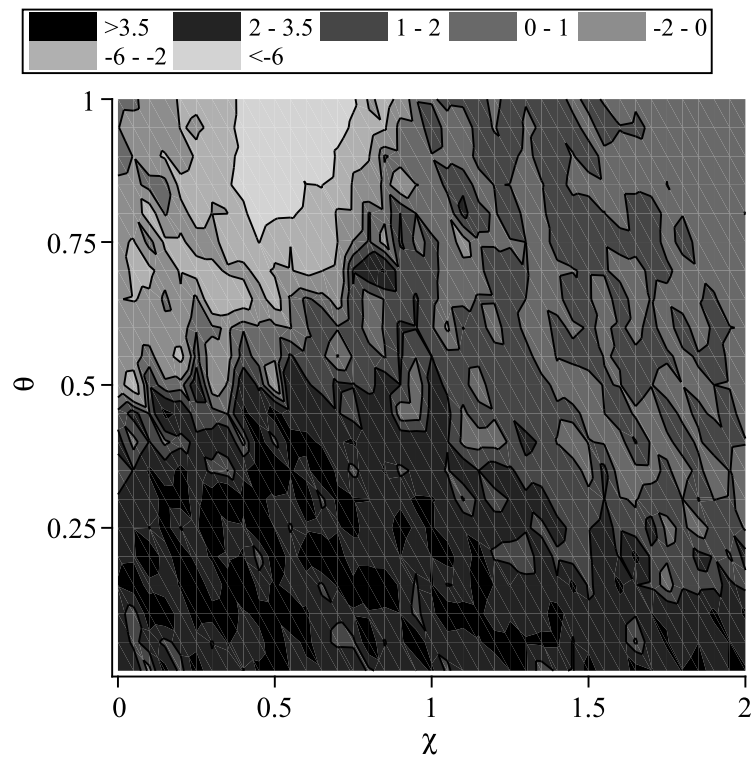
**Fig. 7.** The impact of public efficiency on the Laffer-curve



## References

- Allingham, M. G. and Sandmo, A.: Income Tax Evasion: A Theoretical Analysis. *Journal of Public Economics* 1, (1972) 323–338.
- Antunes, L. B.; Urbano J.; Moniz P. and Roseta-Palma, L. C.: Tax Compliance in a Heterogeneous Multi-agent Society, *Multi-Agent-Based Simulations VI*, Springer (2006), 147–161.
- Bakos, P.; Benczúr, P. and Benedek, D. (2008): “Elasticity of Taxable Wage: Estimates and Flat Tax Predictions Using the Hungarian Tax Changes in 2005”, Budapest, MNB WP 7.
- Bloomquist, K. M.: A Comparison of Agent-Based Models of Income Tax Evasion, *Social Science Computer Review* 24/4. (2006), 411–425.
- Cason, T. N.; Saijo, T. and Yamato, T.: Voluntary Participation and Spite in Public Good Provision Experiments, *Experimental Economics* 5 (2002), 133–153.
- Fehr, E. and Camerer, C. F.: Measuring Social Norms and Preferences Using Experimental Games: A Guide for Social Scientists, in: Heinrich et al. (2004a), 55-96.
- Festinger, L.: A theory of cognitive dissonance. Stanford University Press (1957).
- Frey, B. S. and Weck-Hannemann, H.: The Hidden Economy as an ‘Unobserved’ Variable, *European Economic Review* 26/1-2 (1984), 33–53.
- Frey, B. S. and Torgler, B.: Tax Morale and Conditional Cooperation, *Journal of Comparative Economics* 35/1 (2007), 136–159.
- Garay, B.; Simonovits, A. and Tóth, J. Local interaction in tax evasion, Budapest, Institute of Economics, typescript
- Gigerenzer, G. and Brighton, H.: Homo heuristicus: Why biased minds make better inferences? *Topics in Cognitive Science*, 1 (2009), 107–143.
- Gigerenzer, G. and Selten, R. [eds.]: *Bounded rationality: the adaptive toolbox*. MIT Press (2002).
- Heinrich, J.; Boyd, R.; Bowles, S.; Camerer, C. F.; Fehr, E. and Gintis, H.: *Foundations of Human Sociality*, Oxford University Press (2004).

**Fig. 8.** The difference between the share of tax evaders in a circular and a grid environment



- Heinrich, J.; Boyd, R.; Bowles, S.; Camerer, C. F.; Fehr, E.; Gintis, H. and McElreath, R.: Overview and Synthesis, in: Heinrich et al. (2004a), 8–55.
- Hokamp, S. and Pickhardt, M.: Income Tax Evasion in a Society of Heterogeneous Agents – Evidence from an Agent-based Model, Institute of Spatial and Housing Economics, Working Paper 201035, University of Münster (2010).
- Janssen, M. A. and Ahn, T.K.: Adaptation vs. Anticipation in Public-Good Games, Annual Meeting of the American Political Science Association, Philadelphia (2003).
- Lago-Penas, I. and Lago-Penas, S.: The Determinants of Tax Morale in Comparative Perspective: Evidence from European Countries, *European Journal of Political Economics*, 26(2010) 441–453.
- Meltzoff, A.N.: The Human Infant as Homo Imitans. In: Zentall, T. R. and Galef, B. G. [eds.]: *Social Learning: Psychological and biological perspectives*. Psychology Press (1988), 321–347.
- Prinz, A. (2010): A Moral Theory of Tax Evasion, Working Paper (2010), University of Münster.
- Simon, H. A.: Rational choice and the structure of the environment. *Psychological Review*, Vol. 63 No. 2 (1956), 129–138.
- Simonovits, A.: Tax Morality and Progressive Wage Tax, Discussion Paper 5, Institute of Economics, Hungarian Academy of Sciences (2010).
- Szabó A.; Gulyás L. and Tóth I. J.: The Change in the Share of Tax Evasion: Estimation With the Help of Agent-Based Tax Evasion Simulator TAXSIM, in: Semjén and Tóth (2009), 65–83.
- Tesfatsion, L.: Introduction to the Special Issue on Agent-Based Computational Economics, *Journal of Economic Dynamics and Control* 25(3-4) (2001), 281–293.
- Tesfatsion, L.: Agent-Based Computational Economics: A Constructive Approach to Economic Theory, in: Tesfatsion and Judd (2006), 831–880.
- Tesfatsion, L. and Judd, K L. [eds.]: *Handbook of Computational Economics*, Vol. 2, North Holland (2006).
- Traxler, Ch.: Social Norms and Conditional Cooperative Taxpayers, *European Journal of Political Economy* 26 (2010), 89–103.
- Yitzhaki, S.: Income Tax Evasion: A Theoretical Analysis: A Note, *Journal of Public Economics* 3 (1974), 201–202.